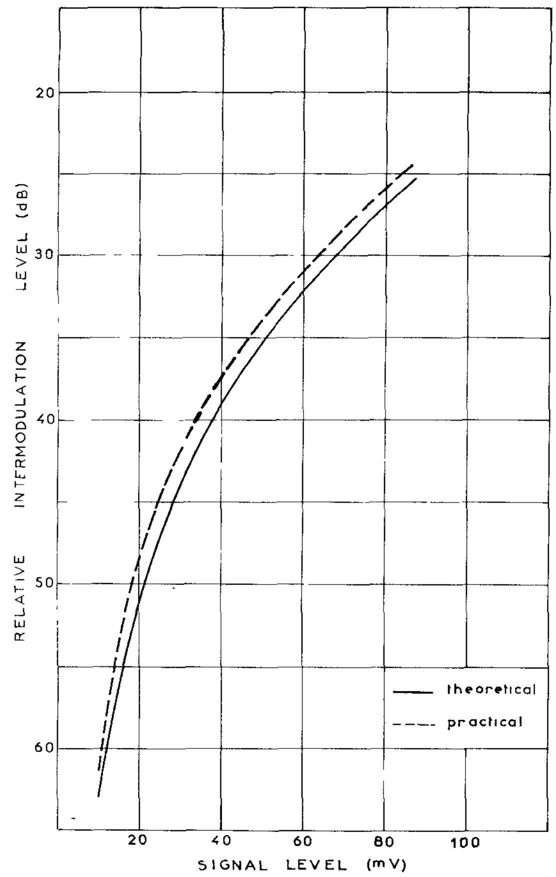

Fig. 1. Distortion performance of up-converter. $B = 1.4 V_1^2 \exp(j140)$; $\gamma = 0.54 \exp(j59.3)$.

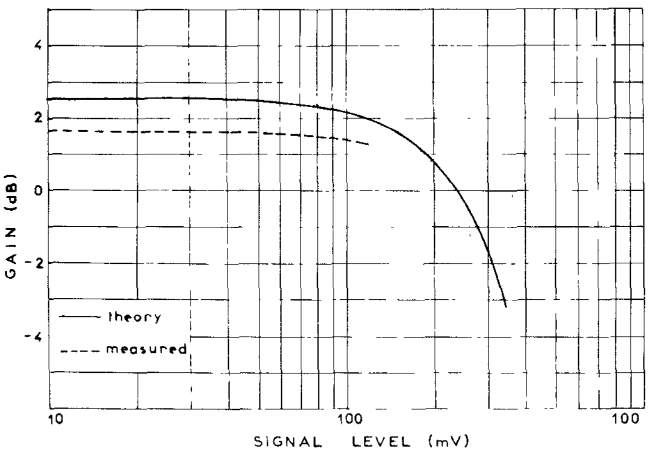

Fig. 2. Variation of gain with signal level.

of distortion. Improvements of 2 dB in gain and 3 dB in distortion level were achieved by increasing the pump voltage from 1 V to 2.2 V.

IV. CONCLUSIONS

It can be concluded that although gain compression and intermodulation distortion are caused by the nonlinearity of the mixing device, one cannot attribute intermodulation distortion to gain compression. This conclusion was confirmed by measurements.

It can also be concluded that gain compression is caused by the generation of a current component at the sideband frequency which is in antiphase with the main sideband current.

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Slant Dielectric Interface Discontinuity in a Waveguide

S. C. KASHYAP

***Abstract*—The reflection and transmission of electromagnetic waves by a slant interface between two dielectric media is investigated. By using suitable Green's functions and a geometrical optics approximation for the field on the dielectric interface, expressions for the transmitted and reflected fields are derived. The approximate results obtained in this manner are compared with the available numerical data and are shown to be fairly accurate for a number of cases of interest.**

INTRODUCTION

Recently, considerable attention has been devoted to the reflection and transmission properties of a slant interface between two dielectric media in a rectangular waveguide. Chow and Wu [1] introduced a moment method with mixed basis functions and applied it to this problem. De Jong and Offringa [2] used suitable waveguide Green's functions to obtain integral representations for the reflected, transmitted, and the unknown field distributions on the interface. Both investigations employed numerical methods for integration. It is the purpose of this short paper to present a much simpler approach by using waveguide Green's functions and a geometrical optics approximation for the field on the interface. In order to obtain this field, the incident field is first divided into two TEM plane waves propagating at angles $\pm\theta_i$ with respect to the waveguide axis. Reflected and transmitted amplitudes of these plane waves, as well as their new directions in each medium, are determined by well-known Fresnel formulas. These plane waves are then utilized to find the fields and their normal derivatives on the interface. Once the fields and their normal derivatives are known, the reflection and transmission coefficients can be determined by using Green's theorem and appropriate Green's functions. The approximate results obtained in this manner are compared with the available numerical data.

THEORY

Consider the slant dielectric discontinuity of Fig. 1 with a TE_{10} mode incident from region I on the interface. The normalized incident electric field is given by

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The author is with the Radio and Electrical Engineering Division, National Research Council of Canada, Ottawa, Ont., Canada.

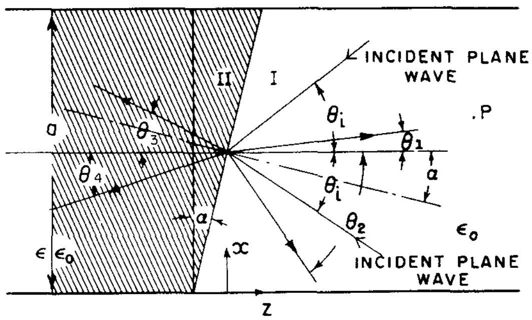

Fig. 1. Slant dielectric interface discontinuity in a waveguide.

$$E_y^i = \sin\left(\frac{\pi x}{a}\right) \exp(-j\beta z) \exp(-j\omega t). \quad (1)$$

The time dependence $\exp(-j\omega t)$ will henceforth be omitted. Here β is the propagation constant for the TE_{10} mode and is given by

$$\beta^2 = k_0^2 - \frac{\pi^2}{a^2}, \qquad k_0 = \frac{2\pi}{\lambda_0} \quad (2)$$

λ_0 being the free-space wavelength.

Applying Green's theorem to region I and choosing the proper Green's function, we obtain the following expression for the reflected field at a point P:

$$\Phi^r(x_p, z_p) = \int_L \{G_1(\boldsymbol{n}\cdot\nabla\Phi) - \Phi(\boldsymbol{n}\cdot\nabla G_1)\}\, ds. \quad (3)$$

Here G_1 is the waveguide Green's function for region I and is given by

$$G_1 = \sum_{m=1}^{\infty} (a\Gamma_{I,m})^{-1} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{m\pi x_p}{a}\right) \cdot \exp(-\Gamma_{I,m}|z - z_p|) \quad (4)$$

with

$$\Gamma_{I,m} = \left[\left(\frac{m\pi}{a}\right)^2 - k_0^2\right]^{1/2}. \quad (5)$$

L represents the boundary between medium I and II and $\boldsymbol{n}$ is a unit vector along the outward (with respect to the region being considered) normal on L. Φ is the reflected field to be evaluated along with its normal derivative on the dielectric interface. To obtain a geometrical approximation for this field, (1) is first rewritten as a sum of two plane waves as follows:

$$E_y^i = \frac{1}{2j}\left\{\exp\left[j\left(\frac{\pi x}{a} - \beta z\right)\right] - \exp\left[-j\left(\frac{\pi x}{a} + \beta z\right)\right]\right\}. \quad (6)$$

Because the interface is slanted at an angle α, these plane waves are incident upon the boundary at different angles given by

$$\theta_{i1} = \theta_i - \alpha \qquad \theta_{i2} = \theta_i + \alpha \quad (7a)$$

where

$$\theta_i = \sin^{-1}\left(\frac{\pi}{k_0 a}\right). \quad (7b)$$

The angles of reflection (or transmission), as well as the magnitudes of the two reflected (or transmitted) waves, will thus be different. Waves of normalized amplitude a_1 and a_2 are reflected at angles θ_1 and θ_2, respectively, where

$$\theta_1 = \theta_i - 2\alpha \qquad \theta_2 = \theta_i + 2\alpha \quad (8)$$

$$a_1 = \frac{\cos\theta_{i1} - (\epsilon - \sin^2\theta_{i1})^{1/2}}{\cos\theta_{i1} + (\epsilon - \sin^2\theta_{i1})^{1/2}} \quad (9a)$$

$$a_2 = \frac{\cos\theta_{i2} - (\epsilon - \sin^2\theta_{i2})^{1/2}}{\cos\theta_{i2} + (\epsilon - \sin^2\theta_{i2})^{1/2}}. \quad (9b)$$

Thus the field Φ to be evaluated on the dielectric interface is given by

$$\Phi = \frac{1}{2j}\{a_1 \exp(jk_0 x \sin\theta_1 + jk_0 z\cos\theta_1) - a_2 \exp(-jk_0 x\sin\theta_2 + jk_0 z\cos\theta_2)\}. \quad (10)$$

The reflected field at a point P in medium I can thus be determined from (3). To obtain the modal reflection coefficients, this field (3) must be represented in terms of waveguide modes; i.e.,

$$\Phi^r(x,z) = \sum_{m=1}^{\infty} R_m \sin\left(\frac{m\pi x}{a}\right)\exp(-\Gamma_{I,m} z) \quad (11)$$

where R_m is the reflection coefficient for the TE_{m0} mode and is given by

$$R_m = (a\Gamma_{I,m})^{-1}\int_L\left\{(\boldsymbol{n}\cdot\nabla\Phi)\sin\left(\frac{m\pi x}{a}\right)\exp(\Gamma_{I,m}z) - \Phi\left[\boldsymbol{n}\cdot\nabla\left(\sin\left(\frac{m\pi x}{a}\right)\exp(\Gamma_{I,m}z)\right)\right]\right\} ds. \quad (12)$$

Substituting (10) for Φ in (12) and simplifying, we obtain

$$R_m = \frac{j\sin\theta_i\tan\theta_m\sec^2\alpha\,(\cos\theta_i + \cos\theta_m)}{2\pi} \cdot\left\{\exp\left(-\frac{j\pi\tan\alpha}{\sin\theta_i}[\cos\theta_i + \cos\theta_m]\right)(-1)^m + 1\right\} \cdot\left[\frac{a_1}{m^2\sin^2\theta_i - [\sin\theta_i - \tan\alpha\,(\cos\theta_i + \cos\theta_m)]^2} - \frac{a_2}{m^2\sin^2\theta_i - [\sin\theta_i + \tan\alpha\,(\cos\theta_i + \cos\theta_m)]^2}\right] \quad (13)$$

where θ_m is the angle at which the plane waves corresponding to the TE_{m0} mode propagate in medium I; i.e.,

$$\sin\theta_m = \frac{m\pi}{k_0 a}. \quad (14)$$

Similar treatment of the transmitted waves in region II leads to the following expression for the transmission coefficient for the TE_{m0} mode:

$$T_m = (-a\Gamma_{II,m})^{-1}\int_L\left\{(\boldsymbol{n}\cdot\nabla\Phi)\sin\left(\frac{m\pi x}{a}\right)\exp(-\Gamma_{II,m}z) - \Phi\left[\boldsymbol{n}\cdot\nabla\left(\sin\left(\frac{m\pi x}{a}\right)\exp(-\Gamma_{II,m}z)\right)\right]\right\} ds \quad (15)$$

with

$$\Gamma_{II,m} = \left[\left(\frac{m\pi}{a}\right)^2 - \epsilon k_0^2\right]^{1/2}. \quad (16)$$

The field Φ to be evaluated along with its normal derivative on the interface is given by

$$\Phi = b_1\exp(jk_0\epsilon^{1/2}x\sin\theta_3 - jk_0\epsilon^{1/2}z\cos\theta_3) - b_2\exp(-jk_0\epsilon^{1/2}x\sin\theta_4 - jk_0\epsilon^{1/2}z\cos\theta_4) \quad (17)$$

with

$$\theta_3 = \alpha + \sin^{-1}\left(\frac{\sin\theta_{i1}}{\epsilon^{1/2}}\right) \qquad \theta_4 = \sin^{-1}\left(\frac{\sin\theta_{i2}}{\epsilon^{1/2}}\right) - \alpha. \quad (18)$$

b_1 and b_2 are the normalized amplitudes of the transmitted plane waves given by

$$b_1 = 1 + a_1 \qquad b_2 = 1 + a_2. \tag{19}$$

After some manipulation, the expression for the transmission coefficient for the TE_{m0} mode simplifies to

$$T_m = \frac{jm \sin^2 \theta_i \sec \phi_m \sec^2 \alpha}{2\pi} \cdot \left[\exp\left(-j\frac{\pi \tan \alpha}{\sin \theta_i} (\cos \theta_i - \epsilon^{1/2} \cos \phi_m) \right) (-1)^m + 1 \right] \cdot \left[\frac{b_1[\cos(\alpha - w_3) + \cos \phi_m]}{m^2 \sin^2 \theta_i - [\sin \theta_i - \tan \alpha (\cos \theta_i - \epsilon^{1/2} \cos \phi_m)]^2} - \frac{b_2[\cos(\alpha + w_4) + \cos \phi_m]}{m^2 \sin^2 \theta_i - [\sin \theta_i + \tan \alpha (\cos \theta_i - \epsilon^{1/2} \cos \phi_m)]^2} \right] \tag{20}$$

with

$$w_3 = \sin^{-1}\left(\frac{\sin \theta_{i1}}{\epsilon^{1/2}}\right) \qquad w_4 = \sin^{-1}\left(\frac{\sin \theta_{i2}}{\epsilon^{1/2}}\right). \tag{21}$$

ϕ_m represents the angle at which the plane waves corresponding to the TE_{m0} mode propagate in medium II, and is given by

$$\sin \phi_m = \frac{m\pi}{\epsilon^{1/2} k_0 a}. \tag{22}$$

An indication of the accuracy of the solution may be obtained by evaluating the ratio of the sum of the reflected and the transmitted powers to the incident power. The expression for this ratio reduces to

$$P_R = \frac{\sum P_s}{P_i} = \frac{\sum \text{reflected powers} + \sum \text{transmitted powers}}{\text{incident power}} = \frac{\sum |R_n|^2 (1 - n^2 \sin^2 \theta_i)^{1/2} + \sum |T_n|^2 (\epsilon - n^2 \sin^2 \theta_i)^{1/2}}{\cos \theta_i}. \tag{23}$$

RESULTS AND DISCUSSION

Figs. 2 and 3 show the results of computations based upon (13), (20), and (23) for one case where $a/\lambda_0 = 1.2$ and $\alpha = 15.94°$. These results have been compared with the available numerical solution [1]. The agreement is found to be quite good for the cases shown. Also, the difference between the sum of scattered powers and the incident power, which is one of the checks on the accuracy of the solution, is found to be less than 0.32 percent. Fig. 3 also indicates that the phase of the reflection coefficient is independent of the change in dielectric constant of the discontinuity. This may be explained by the fact that the direction of the reflected ray only depends upon the slant angle of the discontinuity and not upon its dielectric constant. In Figs. 2 and 3 only the propagating modes up to TE_{30} have been indicated. However, all the propagating modes, the number of which depends upon the dielectric constant, have been considered for the calculation of the power ratio in Fig. 2. The numerical data [1] for $|R_2|$ have been omitted for the sake of clarity. In Fig. 4, the amplitudes of the first two modes in each medium are plotted against the slant angle α. It shows that the amplitude of the dominant modes decreases and that of the higher modes increases with the increase of α. Consequently, more and more power is transferred from the dominant to the higher order modes. Finally, Fig. 5 shows the amplitude of the first two propagating modes in each medium as a function of a/λ_0.

The preceding analysis is not valid for slant angles $\alpha > \pi/6 - \theta_i/3$ because for these cases the plane waves do not follow the simple paths assumed in the analysis, and undergo reflection and transmission at the dielectric boundary more than once. It may be possible to deal with such cases by considering multiple reflections and transmissions at the dielectric interface.

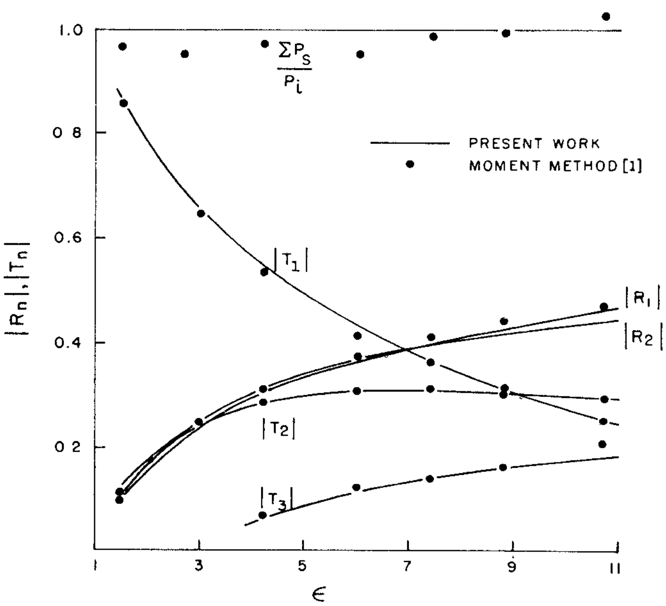

Fig. 2. Magnitudes of transmission and reflection coefficients and sum of normalized powers of the various propagating modes as a function of dielectric constant. TE_{10} mode incidence with $a/\lambda_0 = 1.2$ and $\alpha = 15.94°$ is assumed.

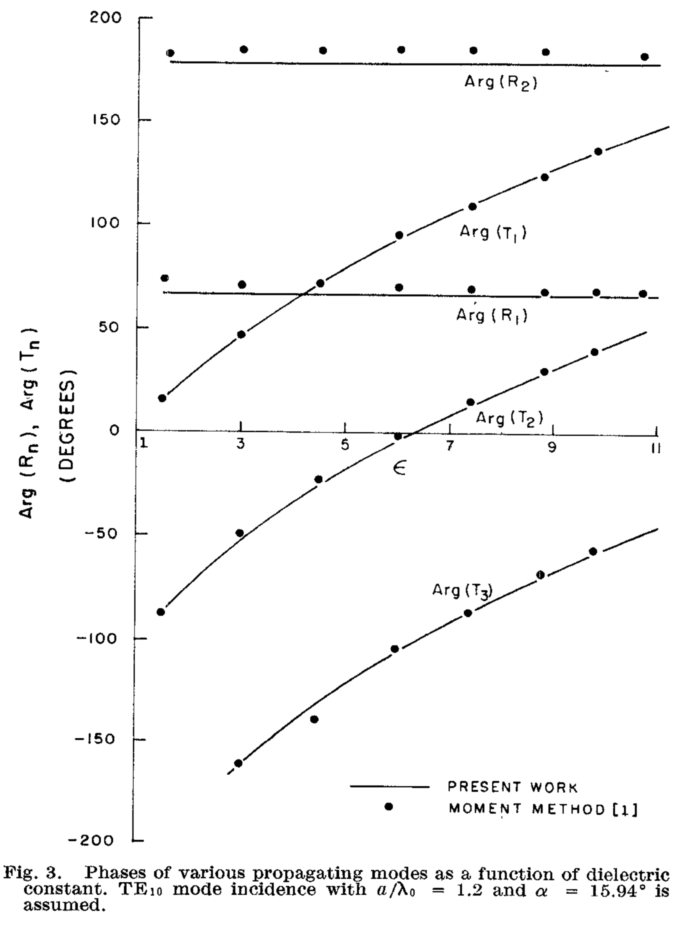

Fig. 3. Phases of various propagating modes as a function of dielectric constant. TE_{10} mode incidence with $a/\lambda_0 = 1.2$ and $\alpha = 15.94°$ is assumed.

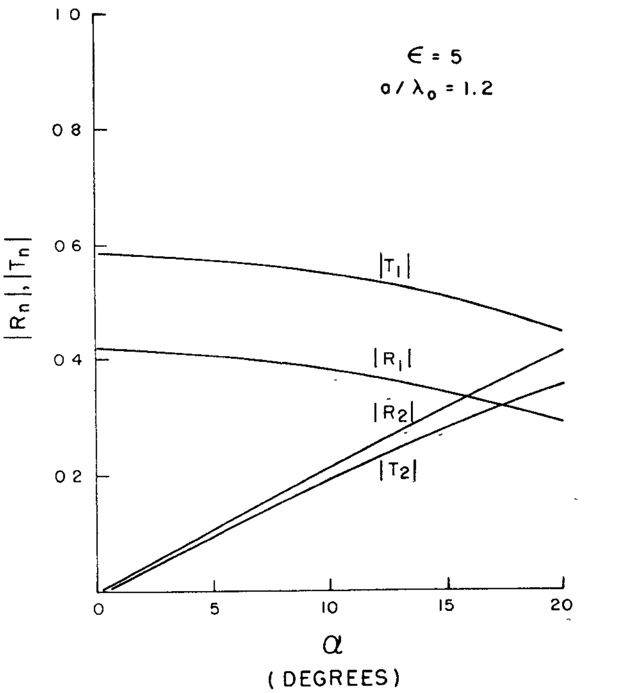

Fig. 4. Amplitudes of first two propagating modes in each medium as a function of slant angle α. TE_{10} mode incidence with $a/\lambda_0 = 1.2$ is assumed.

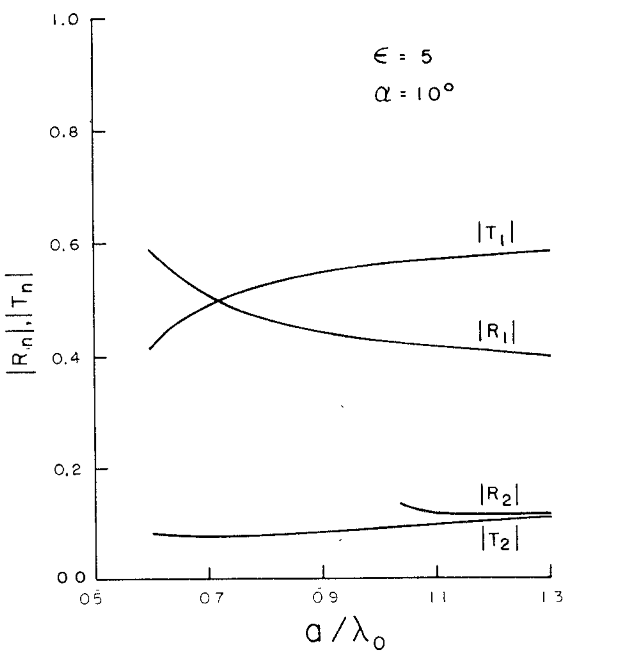

Fig. 5. Amplitudes of first two propagating modes in each medium as a function of a/λ_0.

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The Relations between Scalar Modes in a Lenslike Medium and Vector Modes in a Self-Focusing Optical Fiber

G. L. YIP, MEMBER, IEEE, AND S. NEMOTO

***Abstract*—The relations are established between the scalar modes in an infinite lenslike medium and the vector modes in a self-focusing optical fiber with a finite homogeneous cladding. It is shown that both the transverse fields and the longitudinal fields of the vector modes can be expressed in terms of the scalar modes provided the fiber is operated in the core mode region. Otherwise, significant discrepancies could arise. The scalar modes, however, cannot describe the cladding modes which are caused by the index discontinuity at the outer surface of the cladding.**

I. INTRODUCTION

In integrated and fiber optics many problems involve a medium with an inhomogeneous refractive index. When one is confronted with a problem related to a self-focusing optical fiber, it is essential to know its propagation characteristics. The inhomogeneous nature of this fiber makes its properties more difficult to analyze than the fiber with a homogeneous core. The simplest model of the self-focusing fiber is that of an infinite lenslike medium. For this medium many studies have been carried out using ray-optical method, wave-optics method, and vector field analysis [1], [2]. In particular, the wave-optics method yields the scalar modes. The numerical methods for computing the propagation characteristics of self-focusing fibers with an infinite homogeneous cladding were employed by several authors [3], [4]. Most recently, the numerical method based on an earlier work of Vigants [5] was applied to the more realistic self-focusing fiber with a finite homogeneous cladding [4], [6].

Since the scalar modes can be expressed in analytic functions, it is, in many situations, convenient to approximate the vector modes in the self-focusing fiber by the scalar modes whenever possible. Before doing this, however, it is necessary to understand the relations between the scalar and vector modes and the extent to which the scalar-mode approximation is valid. So far nothing has been reported on this subject. The purpose of this short paper is to establish the relations between the scalar modes in an infinite lenslike medium and the vector modes in a self-focusing fiber with a finite homogeneous cladding, and to show the limitations of the scalar-mode approximation. For this purpose the various propagation characteristics of the scalar modes are compared with those of the vector modes obtained by the numerical method [4], [6].

II. THE SCALAR MODES IN A LENSLIKE MEDIUM

In a cylindrical coordinate system (r,θ,z) the refractive index distribution of the fiber is assumed to be

$$n(r) = \begin{cases} n_0[1 - \Delta(r/a)^2], & 0 \leq r \leq a \\ n_0(1 - \Delta), & a \leq r < b \\ n_3, & b < r < \infty \end{cases} \tag{1}$$

where a and b are the inner and outer radii of cladding, and $0 < \Delta \ll 1$. Let us consider an infinite lenslike medium whose index distribution is expressed as

$$\hat{n}(r) = n_0[1 - (r/d)^2]^{1/2}. \tag{2}$$

The index variation in the core region of the fiber is well approxi-

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The authors are with the Department of Electrical Engineering, McGill University, Montreal, P.Q., Canada.